Verification for Adaptive Mesh Refinement Code

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We present verification and convergence analysis results of adaptive mesh refinement (AMR) calculations for two different AMR implementations. Code verification is extremely important for science-based prediction and simulation. Previous verification focuses on the convergence behavior of uniform grid. With AMR, we can obtain more accurate results with substantially less computational cost. It is assumed in the AMR community that *AMR should achieve the same accuracy in refinement region as the corresponding fine uniform grid* (goal). However, test results show that AMR may not achieve the convergence of equivalent finest uniform grid. In some cases, numerical results with AMR even have larger error than those without AMR. Adaptive Mesh Refinement can also trigger instability for some applications.

We have investigated three model problems. The first two have a smooth solution and the third one contains a shock discontinuity. All of them have exact solutions and represent a variety of problems. We have solved these problems with two hydrodynamics AMR packages: patch-based AMR-MHD [3] and cell-based RAGE AMR. In AMR-MHD, we have tested two different time-step methods: local step where time step varies with the refinement level and locked step where the same stepsize is used for all refinement levels. We have implemented a code to calculate the numerical error for AMR data in different norms.

The first model is a linear wave problem which advects a Gaussian density profile along the diagonal of a rectangular domain. Figure 1 shows that the AMR calculation achieves the same accuracy as the finest uniform grid in AMR-MHD package.

Figure 2 shows the performance of RAGE AMR. We see that only a few AMR calculations achieve better performance in L_∞ error.

The second model problem is about an isentropic vortex propagated along the diagonal of a rectangular domain. It has exactly the same density solution as the linear wave problem. However, since the velocity field and pressure are not constant, the nonlinearity has great impact on the accuracy of numerical solutions. Figure 3 shows the results of AMR-MHD package. We see that AMR with locked step has larger error than with the local step, and even has larger error than the coarse grid without AMR after some time.

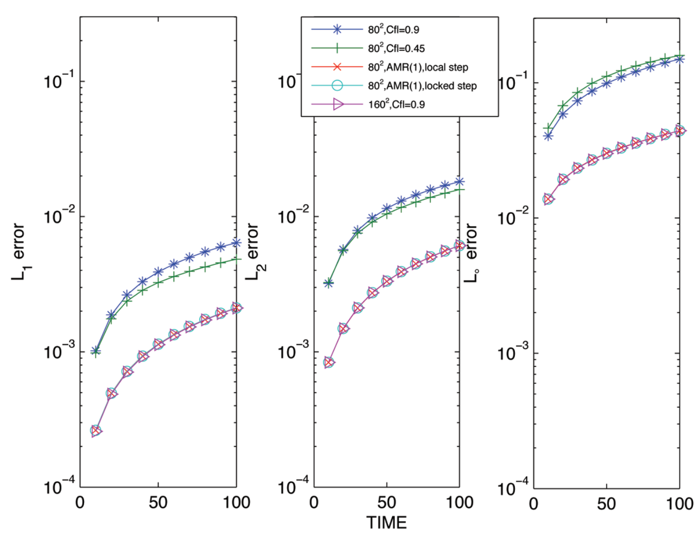

Fig. 1. ***AMR-MHD with local or locked step achieves accuracy of the finest resolution grid.***

For the vortex problem our results with RAGE AMR exhibit greater errors than for the linear wave problem due to the nonlinearity of the problem (see [1] for more detail).

The third model problem is Noh's problem. For a 2-D Noh's problem solved on Cartesian grid, AMR-MHD achieves accuracy of the final resolution grid, whereas RAGE AMR diverges with more refinement levels. A 3-D Noh's problem is solved on *(r,z)* cylindrical grid where we observed a numerical shock instability, *carbuncle* phenomenon, in both AMR results with three or more refinement levels (see Fig. 4). This anomaly was also observed by Gisler [2].

Our comparison of these two AMR codes on these problems has raised issues regarding the effectiveness of RAGE AMR codes in the following areas: a) it has a large initialization error in the first step, b) the refinement criteria do not work well, c) results with AMR are worse than without AMR for high-resolution grid, and d) AMR with more than one-level refinement does not work better than with only one-level refinement. The first issue has been addressed by the code team.

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[1] S. Li, et al., "Two-Dimensional Convergence Study for Problems with Exact Solution: Uniform and Adaptive Grids," Los Alamos National Laboratory report LA-UR-05-7985 (2005).
[2] G. Gisler, "Two-dimensional Convergence Study of the Noh and Sedov Problems with RAGE: Uniform and Adaptive Grids," Los Alamos National Laboratory report LA-UR-05-3207 (2005).
[3] S. Li and H. Li, "A Modern Code for Solving Magneto-hydrodynamic or Hydrodynamic Equations," Los Alamos National Laboratory report LA-UR-03-8926 (December 2003).

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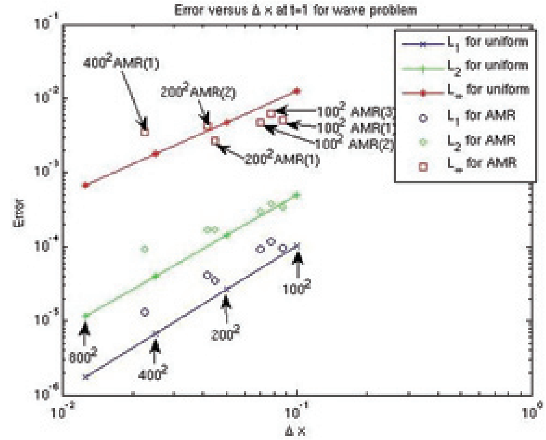

Fig. 2.
Error vs local spacing for the RAGE AMR calculation. The convergence for AMR symbols below the line (uniform grid results) means worse than that of the uniform grid. Two abnormal things: three-level refinement for 100^2 base grid [shown as 100^2AMR(3)] has larger error than one or two level refinement; one-level AMR for 400^2 base grid has larger error than without AMR.

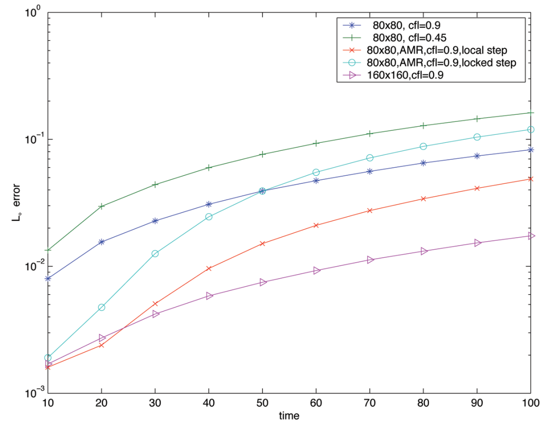

Fig. 3.
AMR with locked step has larger error than with local step, and even larger error than the coarse grid without AMR.

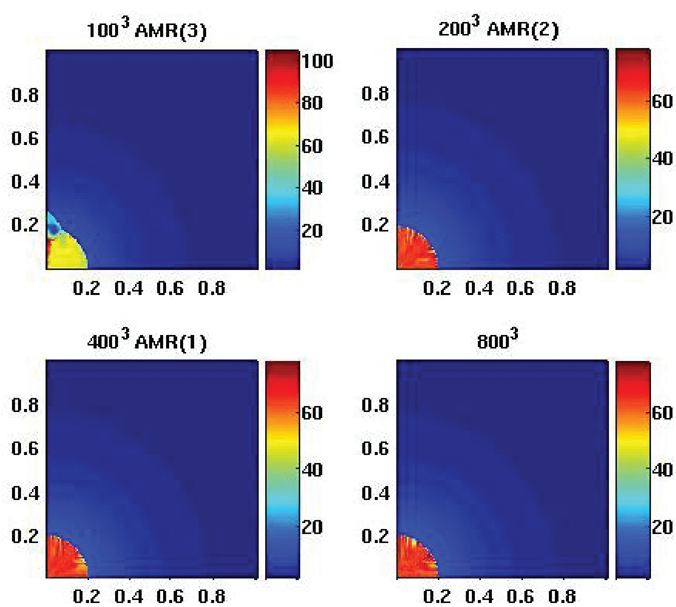

Fig. 4.
Density plot for different refinement levels of the same finest resolutions for Noh's spherical problem. The three-level refinement has a density bubble straddling the shock near r = 0.

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